SOLUTION OF SHORT QUESTIONS

Short Questions

Write the short answers of the following Questions:

Q.1: Prove that:

$$\cos(-\beta) = \cos\beta$$

(IA-2016), (IA-2019)

Sol. L.H.S. $= \cos(-\beta)$

$= \cos(0^\circ - \beta)$

$= \cos 0^\circ \cos\beta + \sin 0^\circ \sin\beta$

$= 1.\cos\beta + 0.\sin\beta$

$= \cos\beta + 0$

$= \cos\beta =$ R.H.S. **Proved.**

Q.2: Prove that:

$$\sin(-\theta) = -\sin\theta$$

(IA-2018), (IA-2021)

Sol. L.H.S. $= \sin(-\theta)$

$= \sin(0^\circ - \theta)$

$= \sin 0^\circ \cos\theta - \cos 0^\circ \sin\theta$

$= 0.\cos\beta - 1.\sin\theta$

$= 0 - \sin\theta$

$= -\sin\theta =$ R.H.S. **Proved.**

Q.3: Prove that: $\tan(-\theta) = -\tan\theta$

Sol. L.H.S. $= \tan(-\theta)$

$= \tan(0^\circ - \theta)$

$$= \frac{\tan 0^\circ - \tan\theta}{1 + \tan 0^\circ \tan\theta}$$

$$= \frac{0 - \tan\theta}{1 + (0)\tan\theta} \qquad \because \left\{\begin{array}{c}\text{Using calculator} \\ \tan 0 = 0\end{array}\right\}$$

$$= \frac{-\tan\theta}{1+0} = -\tan\theta = \text{R.H.S.}$$ **Proved.**

Q.4: Prove that: $\cos\left(\frac{\pi}{2} - \beta\right) = \sin\beta$

(IIA-2016), (IIA-2018), (IA-2021)

Sol. L.H.S. $= \cos\left(\frac{\pi}{2} - \beta\right) = \cos(90^\circ - \beta)$

$= \cos 90^\circ \cos\beta + \sin 90^\circ \sin\beta$

$= (0)\cos\beta + (1)\sin\beta \qquad \because \left\{\begin{array}{c}\text{Using calculator} \\ \cos 90^\circ = 0 \ \& \ \sin 90^\circ = 1\end{array}\right\}$

$= 0 + \sin\beta = \sin\beta =$ R.H.S. **Proved.**

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Q.5: Prove that:

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

(IIA-2021)

Sol. L.H.S. $=\sin\left(\frac{\pi}{2}-\theta\right)$

$=\sin(90^\circ-\theta)$

$=\sin 90^\circ\cos\theta-\cos 90^\circ\sin\theta$

$=(1)\cos\theta-(0)\sin\theta$

$=\cos\theta-0$

$=\cos\theta=$ R.H.S. **Proved.**

Q.6: Prove that:

$$\sin(\pi+\theta)=-\sin\theta$$

Sol. L.H.S $=\sin(\pi+\theta)$

$=\sin(180^\circ+\theta)$

$=\sin 180^\circ\cos\theta+\cos 180^\circ\sin\theta$

$=(0)\cos\theta+(-1)\sin\theta$

$=0-\sin\theta$

$=-\sin\theta=$ R.H.S. **Proved.**

Q.7: Show that: $\sin(\alpha+\beta)+\sin(\alpha-\beta)=2\sin\alpha\cos\beta$

(IIA-2017), (IA-2019), (IIA-2020), (IA-2021)

Sol. L.H.S. $=\sin(\alpha+\beta)+\sin(\alpha-\beta)$

$=\sin\alpha\cos\beta+\cancel{\cos\alpha\sin\beta}+\sin\alpha\cos\beta-\cancel{\cos\alpha\sin\beta}$

$=2\sin\alpha\cos\beta=$ R.H.S. **Proved.**

Q.8: Show that: $\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\sin\alpha\sin\beta$

(IIA-2018)

Sol. L.H.S. $=\cos(\alpha+\beta)-\cos(\alpha-\beta)$

$=(\cos\alpha\cos\beta-\sin\alpha\sin\beta)-(\cos\alpha\cos\beta+\sin\alpha\sin\beta)$

$=\cancel{\cos\alpha\cos\beta}-\sin\alpha\sin\beta-\cancel{\cos\alpha\cos\beta}-\sin\alpha\sin\beta$

$=-2\sin\alpha\sin\beta=$ R.H.S. **Proved.**

Q.9: Prove that: $\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)=\cos\theta$

Sol. L.H.S. $=\sin\left(\theta+\frac{\pi}{6}\right)+\cos\left(\theta+\frac{\pi}{3}\right)$

$=\sin(\theta+30^\circ)+\cos(\theta+60^\circ)$

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$$= \sin\theta\cos 30^\circ + \cos\theta\sin 30^\circ + \cos\theta\cos 60^\circ - \sin\theta\sin 60^\circ$$

$$= \sin\theta\left(\frac{\sqrt{3}}{2}\right) + \cos\theta\left(\frac{1}{2}\right) + \cos\theta\left(\frac{1}{2}\right) - \sin\theta\left(\frac{\sqrt{3}}{2}\right)$$

$$= \cancel{\frac{\sqrt{3}}{2}\sin\theta} + \frac{\cos\theta}{2} + \frac{\cos\theta}{2} - \cancel{\frac{\sqrt{3}}{2}\sin\theta}$$

$$= \frac{\cos\theta}{2} + \frac{\cos\theta}{2} = 2.\frac{\cos\theta}{2} = \cos\theta = \text{R.H.S.}$$ **Proved.**

Q.10: Prove that: $\tan(45^\circ + \theta)\tan(45^\circ - \theta) = 1$

(IA-2017), (IIA-2020)

Sol. L.H.S. $= \tan(45^\circ + \theta)\tan(45^\circ - \theta)$

$$= \frac{\tan 45^\circ + \tan\theta}{1 - \tan 45^\circ\tan\theta} \times \frac{\tan 45^\circ - \tan\theta}{1 + \tan 45^\circ\tan\theta}$$

$$= \frac{1 + \tan\theta}{1 - (1)\tan\theta} \times \frac{1 - \tan\theta}{1 + (1)\tan\theta} \quad \because \left\{\begin{matrix}\text{Using calculator}\\ \tan 45^\circ = 1\end{matrix}\right\}$$

$$= \frac{1 + \tan\theta}{1 - \tan\theta} \times \frac{1 - \tan\theta}{1 + \tan\theta} = 1 = \text{R.H.S.}$$ **Proved.**

Q.11: Express $\sin x\cos 2x - \sin 2x\cos x$ as single term.

(IA-2016), (IIA-2016), (IA-2017), (IA-2018)

Sol. $\sin x\cos 2x - \sin 2x\cos x$

$$= \sin x\cos 2x - \cos x\sin 2x$$

$$= \sin(x - 2x) \quad \because \left\{\begin{matrix}\sin(\alpha-\beta)\\ \sin\alpha\cos\beta - \cos\alpha\sin\beta\end{matrix}\right\}$$

$$= \sin(-x) = \boxed{-\sin x}$$

Q.12: Express $\cos(a + b)\cos(a - b) - \sin(a + b)\sin(a - b)$ as single term. **(IA-2021)**

Sol. $\cos(a + b)\cos(a - b) - \sin(a + b)\sin(a - b)$

$$= \cos\{(a + b) + (a - b)\} \quad \because \left\{\begin{matrix}\cos(\alpha+\beta)\\ = \cos\alpha\cos\beta - \sin\alpha\sin\beta\end{matrix}\right\}$$

$$= \cos(a + b + a - b) = \boxed{\cos 2a}$$

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Q.13: Prove that:

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

(IIA-2019)

Sol. L.H.S. $= \cos 2\alpha$

$= \cos(\alpha+\alpha)$

$= \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$= \cos^2\alpha - \sin^2\alpha = \text{R.H.S.}$

Proved.

Q.14: Prove that:

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$$

(IIA-2016), (IIA-2021)

Sol. L.H.S. $= \tan 2\alpha$

$= \tan(\alpha+\alpha)$

$= \dfrac{\tan\alpha + \tan\alpha}{1-\tan\alpha\tan\alpha}$

$= \dfrac{2\tan\alpha}{1-\tan^2\alpha} = \text{R.H.S.}$ **Proved.**

Q.15: Prove that:

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

(IA-2016)

Sol. R.H.S. $= \dfrac{1-\cos 2\alpha}{2}$

$= \dfrac{1-\cos(\alpha+\alpha)}{2}$

$= \dfrac{1-(\cos\alpha\cos\alpha - \sin\alpha\sin\alpha)}{2}$

$= \dfrac{1-(\cos^2\alpha-\sin^2\alpha)}{2}$

$= \dfrac{1-\cos^2\alpha+\sin^2\alpha}{2}$

$= \dfrac{\sin^2\alpha+\sin^2\alpha}{2} = \dfrac{2\sin^2\alpha}{2}$

$= \sin^2\alpha = \text{L.H.S.}$ **Proved.**

Q.16: Prove that:

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

(IA-2018), (IA-2021)

Sol. R.H.S. $= \dfrac{1+\cos 2\alpha}{2}$

$= \dfrac{1+\cos(\alpha+\alpha)}{2}$

$= \dfrac{1+\cos\alpha\cos\alpha-\sin\alpha\sin\alpha}{2}$

$= \dfrac{1+\cos^2\alpha-\sin^2\alpha}{2}$

$= \dfrac{1-\sin^2\alpha+\cos^2\alpha}{2}$

$= \dfrac{\cos^2\alpha+\cos^2\alpha}{2} = \dfrac{2\cos^2\alpha}{2}$

$= \cos^2\alpha = \text{L.H.S.}$ **Proved.**

Q.17: If $\sin\theta = \dfrac{4}{5}$ and the terminal side of 'θ' lies in 1st quadrant, find $\cos\dfrac{\theta}{2}$. (IA-2017), (IA-2019)

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$$\text{As, } \cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{\frac{25-16}{25}} = \sqrt{\frac{9}{25}} = 3/5$$

Sol.

$$\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{5+3}{5}}{2}} = \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \boxed{\frac{2}{\sqrt{5}}}$$

Q.18: Prove that:

$$\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

(IIA-2018)

Sol. L.H.S. $= \sin\alpha$

$$= \sin\left(\frac{2\alpha}{2}\right)$$

$$= \sin\left(\frac{\alpha+\alpha}{2}\right)$$

$$= \sin\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right)$$

$$= \sin\frac{\alpha}{2}\cos\frac{\alpha}{2} + \cos\frac{\alpha}{2}\sin\frac{\alpha}{2}$$

$$= 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} = \text{R.H.S.}$$

Proved.

Q.19: Prove that:

$$\cos\alpha = \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}$$

Sol. L.H.S. $= \cos\alpha$

$$= \cos\left(\frac{2\alpha}{2}\right)$$

$$= \cos\left(\frac{\alpha+\alpha}{2}\right)$$

$$= \cos\left(\frac{\alpha}{2}+\frac{\alpha}{2}\right)$$

$$= \cos\frac{\alpha}{2}\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\sin\frac{\alpha}{2}$$

$$= \cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2} = \text{R.H.S.}$$

Proved.

Q.20: Express the sum as product: $\cos 12\theta + \cos 4\theta$

(IA-2016), (IA-2020)

Sol. $\cos 12\theta + \cos 4\theta$

$$= 2\cos\left(\frac{12\theta+4\theta}{2}\right)\cos\left(\frac{12\theta-4\theta}{2}\right)$$

$$= 2\cos\left(\frac{16\theta}{2}\right)\cos\left(\frac{8\theta}{2}\right) = \boxed{2\cos 8\theta\cos 4\theta}$$

Q.21: Express $\cos\theta - \cos 4\theta$ **as product.**

(IA-2019), (IA-2022)

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Sol. $\cos\theta - \cos 4\theta$

$$= -2\sin\left(\frac{\theta+4\theta}{2}\right)\sin\left(\frac{\theta-4\theta}{2}\right)$$

$$= -2\sin\left(\frac{5\theta}{2}\right)\sin\left(-\frac{3\theta}{2}\right)$$

$$= \boxed{2\sin\left(\frac{5\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)} \quad \because \{\sin(-\theta) = -\sin\theta\}$$

Q.22: Express as sum or difference of $2\cos 5\theta \sin 3\theta$.

Sol. $2\cos 5\theta \sin 3\theta$

$$= \sin(5\theta + 3\theta) - \sin(5\theta - 3\theta)$$

$$= \boxed{\sin 8\theta - \sin 2\theta}$$

Q.23: Express as sum or difference of $\cos 3\theta \cos\theta$

Sol. $\cos 3\theta \cos\theta$ **(IIA-2016)**

$$= \frac{1}{2}[2\cos 3\theta \cos\theta]$$

$$= \frac{1}{2}[\cos(3\theta+\theta) + \cos(3\theta-\theta)] = \boxed{\frac{1}{2}[\cos 4\theta + \cos 2\theta]}$$

Q.24: Express $\sin(x+30°)+\sin(x-30°)$ as product.

Sol. $\sin(x+30°)+\sin(x-30°)$ **(IA-2018)**

$$= 2\sin\left(\frac{(x+30°)+(x-30°)}{2}\right)\cos\left(\frac{(x+30°)-(x-30°)}{2}\right)$$

$$= 2\sin\left(\frac{x+30°+x-30°}{2}\right)\cos\left(\frac{x+30°-x+30°}{2}\right)$$

$$= 2\sin\left(\frac{2x}{2}\right)\cos\left(\frac{60°}{2}\right) = \boxed{2\sin x \cos 30°}$$